1. If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is :

(2024)

(A) ± 1

(B) $\pm \sqrt{3}$

(C) ± 3

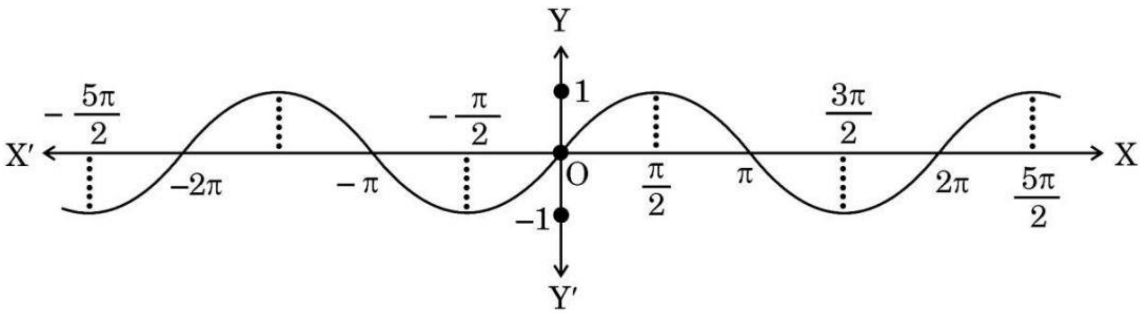
(D) $\pm \frac{1}{3}$

Ans. (D) $\pm \frac{1}{3}$

2. Case Study Based Question : (2024)

If a function f : X → Y defined as f(x) = y is one-one and onto, then we can define a unique function g : Y → X such that g(y) = x, where x ∈ X and y = f(x), y ∈ Y. Function g is called the inverse of function f.

The domain of sine function is R and function sine : R → R is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be defined from set A to [– 1, 1] such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from [– 1, 1] to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval.

(ii) If sin–1 (x) is defined from [– 1, 1] to its principal value branch, find the value of $\sin^{-1}(-1/2) - \sin^{-1}(1)$.

(iii) (a) Draw the graph of $\sin^{-1} x$ from [– 1, 1] to its principal value branch.

(b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$.

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Ans.

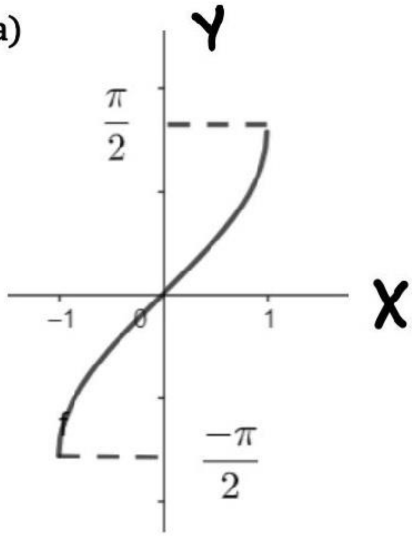
(i) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ or any other interval corresponding to the domain [-1,1]

(ii) $\sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(1)$

$= \frac{-\pi}{6} - \frac{\pi}{2}$

$= \frac{-4\pi}{6}$ or $\frac{-2\pi}{3}$

(iii) (a)



(b) $f(x) = 2\,sin^{-1}(1 - x)$

$-1 \leq 1 - x \leq 1$

$\Rightarrow -2 \leq -x \leq 0$

$\Rightarrow 0 \leq x \leq 2$

Domain $= [0, 2]$

$\frac{-\pi}{2} \leq \sin^{-1}(1 - x) \leq \frac{\pi}{2}$

$-\pi \leq 2\sin^{-1}(1 - x) \leq \pi$

So range $= [-\pi, \pi]$

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Previous Years' CBSE Board Questions

2.2 Basic Concepts

MCQ

1. $\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to
   (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$ *(2023)*

2. If $f(x) = |\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is
   (a) 1 (b) −1 (c) $\frac{-1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$ *(2023)*

3. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.
   Assertion (A) : All trigonometric functions have their inverses over their respective domains.
   Reason (R) : The inverse of $\tan^{-1}x$ exists for some $x \in \mathbb{R}$.
   (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
   (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
   (c) Assertion (A) is true but Reason (R) is false.
   (d) Assertion (A) is false but Reason (R) is true. *(2023)*

4. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is
   (a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$ *(Term I, 2021-22)* U

5. The principal value of $\cot^{-1}(-\sqrt{3})$ is
   (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$ *(2020)*

6. $\tan^{-1}3 + \tan^{-1}\lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ?
   (a) $\lambda \in \left(-\frac{1}{3},\frac{1}{3}\right)$ (b) $\lambda > \frac{1}{3}$
   (c) $\lambda < \frac{1}{3}$ (d) All real values of λ *(2020)*

7. The principal value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ is
   (a) $\frac{2\pi}{5}$ (b) $\frac{-2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) $\frac{-3\pi}{5}$ *(2020)* U

VSA (1 mark)

8. The range of the principal value branch of the function $y = \sec^{-1}x$ is ________. *(2020)*

9. The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is ________. *(2020)* U

10. Write the value of $\cos^{-1}\left(-\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$. *(Foreign 2014)* Ap

11. Write the principal value of $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$. *(AI 2014C)*

12. Find the value of the following :
    $\cot\left(\frac{\pi}{2}-2\cot^{-1}\sqrt{3}\right)$ *(AI 2014C)*

SA I (2 marks)

13. Write the domain and range (principle value branch) of the following functions:
    $f(x) = \tan^{-1}x$. *(2023)*

14. Evaluate : $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$ *(2023)*

15. Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$. *(2021C)*

16. Prove that : $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ *(2020)* Ev

17. Prove that : $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\cos^{-1}x, \frac{1}{\sqrt{2}}\le x\le 1$. *(2020)* Ap

LA I (4 marks)

18. Solve for x : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ *(2020C)*

19. Prove that : $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$. *(2020C)* Ap

20. Prove that : $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in[0,1]$ *(2020, 2019C)*

21. Prove that :
    $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$ *(AI 2019)* Ev

22. Prove that : $\sin^{-1}\frac{4}{5}+\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{63}{65}=\frac{\pi}{2}$ *(2019)*

23. If $\tan^{-1}x-\cot^{-1}x=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$, find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$. *(2019)* Ev

24. Find the value of $\sin\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$ *(2019)*

25. Prove that :
    $\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x; -\frac{1}{\sqrt{2}}\le x\le 1$ *(2019C)* Ap

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CBSE Sample Questions

2.2 Basic Concepts

MCQ

1. In the given question, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
Assertion (A) : The domain of the function $\sec^{-1} 2x$ is
$\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$
Reason (R) : $\sec^{-1}(-2) = -\frac{\pi}{4}$
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true. *(2022-23)*

2. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) -1 (d) 1
(Term I, 2021-22) Ap

3. $\sin(\tan^{-1}x)$, where $|x| < 1$, is equal to
(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$
(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$
(Term I, 2021-22)

4. Simplest form of
$\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right), \pi < x < \frac{3\pi}{2}$ is
(a) $\frac{\pi}{4} - \frac{x}{2}$ (b) $\frac{3\pi}{2} - \frac{x}{2}$ (c) $-\frac{x}{2}$ (d) $\pi - \frac{x}{2}$
(Term I, 2021-22) Cr

5. If $\tan^{-1} x = y$, then
(a) $-1 < y < 1$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
(c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $y \in \left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$
(Term I, 2021-22)

SA I (2 marks)

6. Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$. *(2022-23)*

7. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. *(2020-21)* Ev

Detailed SOLUTIONS

1. **(a) :** We have,
$$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3}+\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right] = \sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]$$
$$= \sin\left(\frac{\pi}{2}\right) = 1$$

2. **(d) :** $f(x) = |\cos x|$
At $\frac{\pi}{2} < x < \pi$, $\cos x < 0$ $\therefore |\cos x| = -\cos x \Rightarrow f(x) = -\cos x$
$$\therefore f\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{3\pi}{4}\right) = -\cos\left(\pi - \frac{\pi}{4}\right)$$
$$= \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad [\because \cos(\pi-\theta) = -\cos\theta]$$

3. **(d) :** All trigonometric functions are periodic and hence not invertible over their respective domains but all trigonometric functions have inverse over their restricted domains.
Inverse of $\tan^{-1}x$ is $\tan x$ which is defined for
$$x \in R - (2n+1)\frac{\pi}{2}, n \in Z$$
$\therefore$ Assertion is false and reason is true.

4. **(b) :** We have, $\sin^{-1}\left(\cos\frac{13\pi}{5}\right) = \sin^{-1}\left[\cos\left(2\pi + \frac{3\pi}{5}\right)\right]$
$$= \sin^{-1}\left[\cos\frac{3\pi}{5}\right] = \sin^{-1}\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{10}\right)\right]$$
$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) = -\sin^{-1}\left(\sin\frac{\pi}{10}\right) = -\frac{\pi}{10}$$

Answer Tips
- $\cos(2\pi + \theta) = \cos\theta$, $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

5. **(d) :** We know that $\cot^{-1}(x) \in (0, \pi)$
$$\cot^{-1}(-\sqrt{3}) = \cot^{-1}\left(-\cot\frac{\pi}{6}\right)$$
$$= \cot^{-1}\left[\cot\left(\pi - \frac{\pi}{6}\right)\right] \quad [\because \cot(\pi-\theta) = -\cot\theta]$$
$$= \cot^{-1}\left[\cot\left(\frac{5\pi}{6}\right)\right] = \frac{5\pi}{6} \quad [\because \cot^{-1}[\cot\theta] = \theta]$$
Thus, the principal value of $\cot^{-1}(-\sqrt{3})$ is $\frac{5\pi}{6}$.

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6. (c) : Given, $\tan^{-1}3+\tan^{-1}\lambda=\tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$

$\tan^{-1}3+\tan^{-1}\lambda=\tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ for $3\lambda<1$ $\therefore$ $3\lambda<1\Rightarrow\lambda<\frac{1}{3}$

Concept Applied

- $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy<1$

7. (b) : We have, $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$

We know that the range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)=\tan^{-1}\left(\tan\left(\pi-\frac{2\pi}{5}\right)\right)$

$=\tan^{-1}\left[-\tan\left(\frac{2\pi}{5}\right)\right]$ $[\because\ \tan(\pi-\theta)=\tan\theta]$

$=-\tan^{-1}\left[\tan\left(\frac{2\pi}{5}\right)\right]=-\frac{2\pi}{5}$ $[\because\ \tan^{-1}(\tan\theta)=\theta]$

8. The range of the principal value branch of the function $y=\sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

9. Let $y=\cos^{-1}\left(\frac{-1}{2}\right)\Rightarrow\cos y=\frac{-1}{2}\Rightarrow\cos y=-\cos\left(\frac{\pi}{3}\right)$

Since, the range of $\cos^{-1}x$ is $[0,\pi]$

$\Rightarrow$ $\cos y=-\cos\left(\pi-\frac{\pi}{3}\right)$ $[\because\ \cos(\pi-\theta)=-\cos\theta]$

$\Rightarrow$ $y=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

Hence, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

10. Given $\cos^{-1}\left(-\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

$=\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+2\sin^{-1}\left(\sin\frac{\pi}{6}\right)=\frac{2\pi}{3}+2\times\frac{\pi}{6}=\pi$

[$\because$ Range of $\cos^{-1}x$ is $[0,\pi]$ & of $\sin^{-1}x$ is $[-\pi/2,\pi/2]$]

Commonly Made Mistake

- Remember the domain and range of inverse trigonometric functions.

11. Here, $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]=\tan^{-1}(-1)=-\frac{\pi}{4}$

This is the required principal value as it is lie in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Key Points

- The range of $y=\tan^{-1}x$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

12. $\cot\left(\frac{\pi}{2}-2\cot^{-1}\sqrt{3}\right)$

$=\cot\left(\frac{\pi}{2}-2\cot^{-1}\left(\cot\frac{\pi}{6}\right)\right)=\cot\left(\frac{\pi}{2}-2\cdot\frac{\pi}{6}\right)$

$=\cot\left(\frac{\pi}{2}-\frac{\pi}{3}\right)=\cot\frac{\pi}{6}=\sqrt{3}$

13. Domain of $\tan^{-1}x=(-\infty,\infty)$ and range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

14. $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]=\cos^{-1}\left[\cos\left(\frac{7\pi}{3}\right)\right]$ $(\because\ \cos(-\theta)=\cos\theta)$

$=\cos^{-1}\left(\cos\left(2\pi+\frac{\pi}{3}\right)\right)=\cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$

$=\frac{\pi}{3}$ $(\because\ \cos^{-1}(\cos x)=x\ \forall\ 0\le x\le\pi)$

15. Let $x=\cos\theta$

$\sec^{-1}\left(\frac{1}{2x^2-1}\right)=\sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right)=\sec^{-1}\left(\frac{1}{\cos2\theta}\right)$

$=\sec^{-1}(\sec2\theta)=2\theta$

Hence, $\sec^{-1}\left(\frac{1}{2x^2-1}\right)=2\cos^{-1}x$

Concept Applied

- $\cos2\theta=2\cos^2\theta-1$, $\sec\theta=\frac{1}{\cos\theta}$

16. Consider L.H.S. $=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{3}\right)\right]$

$=\frac{9}{4}\cos^{-1}\left(\frac{1}{3}\right)$ ...(i) $\left[\because\ \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right]$

Let $a=\cos^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow$ $\cos a=\frac{1}{3}\Rightarrow\sin a=\sqrt{1-\cos^2a}$ $[\because\ \sin^2\theta+\cos^2\theta=1]$

$\Rightarrow$ $\sin a=\sqrt{1-\left(\frac{1}{3}\right)^2}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}\Rightarrow$ $a=\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

So, L.H.S. $=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=$ R.H.S.

17. Consider L.H.S. $=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x=\cos\theta$, we get

$=\sin^{-1}\left(2\cos\theta\sqrt{\sin^2\theta}\right)$ $[\because\ \sin^2\theta+\cos^2\theta=1]$

$=\sin^{-1}(2\cos\theta\sin\theta)=\sin^{-1}(\sin2\theta)$

$=2\theta$ $[\because\ \sin^{-1}(\sin\theta)=\theta]$

Since, $x=\cos\theta$

$\Rightarrow$ $\theta=\cos^{-1}x$

$\therefore$ $2\theta=2\cos^{-1}x=$ R.H.S.

18. Given, $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

Put $x=\sin y$

$\Rightarrow$ $\sin^{-1}(1-\sin y)-2\sin^{-1}(\sin y)=\frac{\pi}{2}$

$\Rightarrow$ $\sin^{-1}(1-\sin y)-2y=\frac{\pi}{2}$ $[\sin^{-1}(\sin\theta)=\theta]$

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$\Rightarrow \quad \sin^{-1}(1-\sin y)=\frac{\pi}{2}+2y \Rightarrow 1-\sin y=\sin\left(\frac{\pi}{2}+2y\right)$

$\Rightarrow \quad 1-\sin y=\cos 2y \qquad [\sin(\pi/2+\theta)=\cos\theta]$

$\Rightarrow \quad 1-\sin y=1-2\sin^2 y \qquad [\because \cos 2\theta=1-2\sin^2\theta]$

$\Rightarrow \quad 2\sin^2 y-\sin y=0$

Replace $\sin y = x$

$\Rightarrow \quad 2x^2-x=0 \Rightarrow x(2x-1)=0 \Rightarrow x=0, \frac{1}{2}$

19. Consider, L.H.S. $= 2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$

$=\tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\frac{1}{7} \quad =\tan^{-1}\frac{1}{\frac{3}{4}}+\tan^{-1}\frac{1}{7}$

$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7} \quad =\tan^{-1}\frac{\left(\frac{4}{3}+\frac{1}{7}\right)}{1-\frac{4}{21}}=\tan^{-1}\frac{\frac{31}{21}}{\frac{17}{21}}$

$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$=\tan^{-1}\frac{31}{17}=\text{R.H.S.}$

Hence proved.

Concept Applied

- $2\tan^{-1}\theta=\tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right)$

20. Consider, R.H.S. $= \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x=\tan^2\theta$ ...(i)

$\therefore \quad \text{R.H.S.} = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)=\frac{1}{2}\cos^{-1}(\cos 2\theta)=\frac{1}{2}(2\theta)=\theta$

From equation (i), we get

$\tan\theta=\sqrt{x} \Rightarrow \theta=\tan^{-1}\sqrt{x} = \text{L.H.S.}$

$\therefore \quad$ L.H.S. = R.H.S.

Hence proved.

21. Let $x=\cos^{-1}\left(\frac{12}{13}\right)$ and $y=\sin^{-1}\left(\frac{3}{5}\right)$

or $\quad \cos x=\frac{12}{13}$ and $\sin y=\frac{3}{5}$

Now, $\sin x=\sqrt{1-\cos^2 x}$ and $\cos y=\sqrt{1-\sin^2 y}$

$\Rightarrow \quad \sin x=\sqrt{1-\frac{144}{169}}$ and $\cos y=\sqrt{1-\frac{9}{25}}$

$\Rightarrow \quad \sin x=\frac{5}{13}$ and $\cos y=\frac{4}{5}$

We know that,

$\sin(x+y)=\frac{5}{13}\times\frac{4}{5}+\frac{12}{13}\times\frac{3}{5}=\frac{20}{65}+\frac{36}{65}=\frac{56}{65} \Rightarrow x+y=\sin^{-1}\left(\frac{56}{65}\right)$

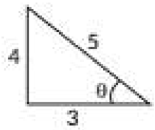
or, $\quad \cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$

Concept Applied

- $\sin(x+y) = \sin x\cos y + \cos x\sin y$

22. Consider L.H.S. $= \sin^{-1}\frac{4}{5}+\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{63}{65}$

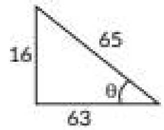
$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{63}{65}$



$=\tan^{-1}\left[\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}\right]+\cos^{-1}\frac{63}{65}$

$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$=\tan\left[\frac{63}{16}\right]+\tan^{-1}\left[\frac{16}{63}\right]$



$=\tan^{-1}\left[\frac{\frac{63}{16}+\frac{16}{63}}{1-\frac{63}{16}\times\frac{16}{63}}\right] = \tan^{-1}(\infty) = \frac{\pi}{2}=\text{R.H.S.}$

Hence proved.

23. Given, $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad \tan^{-1}x - \tan^{-1}\left(\frac{1}{x}\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\left[\because \tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}x, x>0\right]$

$\Rightarrow \quad \tan^{-1}\left(\frac{x-\frac{1}{x}}{1+x\cdot\frac{1}{x}}\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\left[\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left[\frac{x-y}{1+xy}\right]\right]$

$\Rightarrow \quad \frac{x^2-1}{2x}=\frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}x^2-\sqrt{3}=2x \Rightarrow \sqrt{3}x^2-2x-\sqrt{3}=0$

$\Rightarrow \quad \sqrt{3}x^2-3x+x-\sqrt{3}=0 \Rightarrow \sqrt{3}x(x-\sqrt{3})+1(x-\sqrt{3})=0$

$\Rightarrow \quad (\sqrt{3}x+1)(x-\sqrt{3})=0 \Rightarrow \sqrt{3}x+1=0$ or $x-\sqrt{3}=0$

$\Rightarrow \quad x=\frac{-1}{\sqrt{3}}$ or $x=\sqrt{3}$

Since, $x > 0$

So, $x=\frac{-1}{\sqrt{3}}$ is rejected. $\therefore \quad \sec^{-1}\left(\frac{2}{x}\right)=\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\frac{\pi}{6}$

24. We have, $\sin\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$

Let $\cos^{-1}\frac{4}{5}=A \Rightarrow \cos A=\frac{4}{5}$

We know that $\sin A=\sqrt{1-\cos^2 A}$

$\Rightarrow \quad \sin A=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

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Let $\tan^{-1}\frac{2}{3}=B \Rightarrow \tan B=\frac{2}{3}$

We know that, $1+\tan^2\theta=\sec^2\theta$

$\Rightarrow \quad \sec B=\sqrt{1+\tan^2 B}=\sqrt{1+\frac{4}{9}}=\frac{\sqrt{13}}{3}$

$\Rightarrow \quad \cos B=\frac{3}{\sqrt{13}} \qquad \left[\because \sec\theta=\frac{1}{\cos\theta}\right]$

$\Rightarrow \quad \sin B=\sqrt{1-\cos^2 B}=\sqrt{1-\frac{9}{13}}=\frac{2}{\sqrt{13}}$

Now, $\sin\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)=\sin(A+B)$

$=\sin A\cos B+\cos A\sin B$

$[\because \sin(x+y)=\sin x\cos y+\cos x\sin y]$

$=\frac{3}{5}\times\frac{3}{\sqrt{13}}+\frac{4}{5}\times\frac{2}{\sqrt{13}}=\frac{9}{5\sqrt{13}}+\frac{8}{5\sqrt{13}}=\frac{17}{5\sqrt{13}}$

25. Consider L.H.S. $=\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)$

Put $x=\cos\theta$, we get

L.H.S. $=\tan^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right]$

$=\tan^{-1}\left[\frac{\left|\sqrt{2}\cos\frac{\theta}{2}\right|+\left|\sqrt{2}\sin\frac{\theta}{2}\right|}{\left|\sqrt{2}\cos\frac{\theta}{2}\right|-\left|\sqrt{2}\sin\frac{\theta}{2}\right|}\right]$

$[\because 1+\cos 2\theta=2\cos^2\theta,\ 1-\cos 2\theta=2\sin^2\theta]$

$=\tan^{-1}\left[\frac{-\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}{-\sqrt{2}\cos\frac{\theta}{2}-\sqrt{2}\sin\frac{\theta}{2}}\right]=\tan^{-1}\left[\frac{-\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{-\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right]$

$=\tan^{-1}\left[\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right]=\tan^{-1}\left[\frac{\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}{\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}\right]=\tan^{-1}\left[\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right]$

$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right]=\frac{\pi}{4}-\frac{\theta}{2} \qquad [\because \tan^{-1}(\tan\theta)=\theta]$

Since, $x=\cos\theta \Rightarrow \theta=\cos^{-1}x$

$\therefore \quad$ L.H.S. $=\frac{\pi}{4}-\frac{\cos^{-1}x}{2}=$ R.H.S.

CBSE Sample Questions

1. (c) : $\sec^{-1}x$ is defined if $x\le -1$ or $x\ge 1$.

Hence, $\sec^{-1}2x$ will be defined if $x\le -\frac{1}{2}$ or $x\ge\frac{1}{2}$

The range of the function $\sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ (1)

Hence, A is true and R is false.

2. (d) : We have,

$\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$

$=\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]=\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]=\sin\left(\frac{\pi}{2}\right)=1$ (1)

3. (d) : We have, $\sin(\tan^{-1}x)$

Let $\tan^{-1}x=\theta \Rightarrow x=\tan\theta \Rightarrow \sin\theta=\frac{x}{\sqrt{x^2+1}}$

$\therefore \quad \sin(\tan^{-1}x)=\sin\theta=\frac{x}{\sqrt{x^2+1}}$ (1)

4. (a) : We have,

$\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right),\ \pi<x<\frac{3\pi}{2}$

$=\tan^{-1}\left(\frac{\left|\sqrt{2}\cos\frac{x}{2}\right|+\left|\sqrt{2}\sin\frac{x}{2}\right|}{\left|\sqrt{2}\cos\frac{x}{2}\right|-\left|\sqrt{2}\sin\frac{x}{2}\right|}\right)$

$=\tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2}+\sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos\frac{x}{2}-\sqrt{2}\sin\frac{x}{2}}\right) \quad \left(\because \frac{\pi}{2}<\frac{x}{2}<\frac{3\pi}{4}\right)$

$=\tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)=\tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\frac{\pi}{4}-\frac{x}{2}$ (1)

5. (c) : Range of $\tan^{-1}x=\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\therefore \quad \frac{-\pi}{2}<y<\frac{\pi}{2}$ (1)

6. Given, $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]=\sin^{-1}\left[\sin\left(2\pi-\frac{\pi}{7}\right)\right]$ (1)

$=\sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right]=-\frac{\pi}{7}$ (1)

7. We have,

$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ (1/2)

$=\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$

$=\tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]=\tan^{-1}\left[\tan\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}\right]$ (1)

$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]=\frac{\pi}{4}+\frac{x}{2}$ (1/2)

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